Why not buy lottery tickets?

Beyond "you can't win if you don't play"

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Everyone knows buying lottery tickets is a bad idea. But what does that mean? Sure, you will most likely lose your dollar. On the other hand, there's a small chance that you will win big. If you view playing the lottery as a gamble, like playing roulette, then you are probably playing for fun and you don't mind losing your dollar. But what if you really want to make money? Can lottery tickets be a good investment? And if so, how should you buy them in order to make the best investment? Should you wait until the jackpot is "big enough'' and then buy tickets? How big is big enough?

Chances are our instinct is the same as yours: there is no way a lottery ticket could be a good investment. Even if the chance of winning weren't astronomically small, the fact remains that the lottery operators are running a profitable business. If they make money, then the players lose money.

We are mathematicians, however. When large lottery prizes are announced, we get these kinds of questions from our relatives. They also come up in the classroom when we teach probability, a mathematical field founded centuries ago to study gambling. In order to face our families and students —and keep our dignity and self-respect!—we decided to justify our instincts with solid mathematics.

What we found is that mathematical techniques alone are not enough to answer these questions.

Some mathematics

Let's make a brief comparison between the lottery and roulette. When you bet a dollar on a spin of a roulette wheel, you can expect to win back around ninety-five cents on average. The five cents you lose is how casinos make their money. The same is true for lotteries, except that in typical lotteries you can expect to get about fifty-five cents out of a dollar ticket. Again, this is on average: in both roulette and the lottery, there is a high probability that you will lose any particular game. There are two main differences between these two games. The first is the scale of the numbers: the lottery jackpot is much larger than a winning roulette payout, but the chance of winning is correspondingly much smaller. But the second difference is what makes things interesting from a mathematical point of view. To the gambler, any spin of the roulette wheel is equivalent to any other: the odds and payouts are the same. But this is not true of lottery

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drawings because the jackpot changes from week to week.

The format for lotteries like Mega Millions is that you pick, say, six numbers. The prize you win depends on how many of the six winning numbers you match. The players who match all six numbers split a large prize called the jackpot. (The smaller prizes for matching some but not all of the numbers help the gambler a little.) If no one matches all six numbers in a particular drawing, then the jackpot is retained and increased for the next drawing. Consequently, if no one wins the jackpot for several drawings in a row, the prize can become truly enormous. In March 2007, the jackpot for Mega Millions was announced at $390 million. Since there are only 176 million possible ways to pick the six numbers, it looks like this is a great bet: one dollar is cheap for a 1-in-176 million chance at winning $390 million. On average, it would seem that a lottery ticket for that game was worth about $2.21.

But there are several problems with this. First, the jackpot amount "$390 million" that you see on billboards is not the amount of cash you actually stand to win. Rather, it is the sum of various future annuity payments. The amount you can actually win as cash is substantially lower, around 60 percent. Second, lottery winnings are considered income and are taxed at the federal and state level, so subtract at least another 25 percent from the advertised value. This takes the $390 million jackpot down to (coincidentally) about $176 million.

There is still another problem—someone else might also win the jackpot. And if multiple people have winning tickets, they split the jackpot evenly. That is what happened with the $390 million jackpot, for example, which was shared by a truck driver from Rocky Face, Georgia, and someone from New Jersey. So although a very large jackpot seems good for the gambler, it will sell more tickets and increase the chance of sharing the jackpot, which is bad for the gambler. Because of these contradictory forces, it is not entirely straightforward to calculate when the average rate of return on a lottery ticket is positive. This is presumably why different people have different ideas about a "smart" strategy for buying lottery tickets.

With a little elementary calculus, however, it is not a hard problem to solve. In the $390 million Mega Millions drawing, 212 million tickets were sold, which is more than the number of possible ways to pick the six numbers (176 million), so it may come as no surprise that in this case the danger of sharing the jackpot made the drawing a bad bet. In fact, we discovered that all Mega Millions drawings have been bad bets, essentially because so many tickets are sold for each drawing.

In principle, though, the jackpot can get so large that a lottery ticket offers, on average, a positive rate of return. That is unlikely to ever happen, however. We made a mathematical model for Mega Millions and its competitor, Powerball, which shows that in all likelihood these lotteries will never offer a good bet. That is, as the jackpot grow and more tickets are sold, enough extra tickets are sold to nullify the benefit of the bigger jackpot.

But there is more to the story. The huge jackpots in Mega Millions drawings get a lot of press, but the smaller single-state lotteries like Georgia's Fantasy 5 offer better rates of return. In fact, to our great surprise, in some cases single-state lotteries have had positive rates of return, even as large as 30 percent. That is, for these drawings a dollar ticket would give you back $1.30, on average. We did not expect this! Our models show that this is due to a larger ratio of jackpot size to total number of tickets sold for single-state lotteries.

If you figure that buying a lottery ticket is like making an investment on the time frame of a week, then a 30 percent rate of return is fantastic. In comparison, the stock market has historically returned about 0.1 percent to 0.15 percent on average over a week, and lots of people invest in stocks. Returns of 30 percent in a week's time are what you might hope for from highly risky investments, such as oil speculation or the lottery.

Help! Some economics

So why not buy lottery tickets instead of stocks? Certainly the instinctive answer is because you still won't win. The technical word for this is risk. It's true that the high rate of return is only an average for all lottery tickets for a particular drawing, and most people in that drawing won't win the jackpot.

What we need is a way to compare investments with various levels of return and risk. You could put your money in a savings account with no risk at all but a tiny rate of return. Or you could buy stocks, which have moderate risk and a moderate rate of return. Many people choose to buy stocks, so sometimes people will take a riskier investment if it promises a better reward.

So what about the lottery tickets with mind-boggling levels of risk and return? Should you invest? Sadly, mathematics doesn't provide the answer to this question. Luckily, however, basic economics does.

In the 1950s and 60s, economists developed what is sometimes called modern portfolio theory. Pioneered by Harry Markowitz, who won a Nobel Prize for the idea, this field explains how to distribute your money among investments with different rates of return and different levels of risk. This principle is now taught to Emory undergraduates in Business 423. You have to input a few things: the returns and risks associated with each investment and how they are correlated with each other. All of these things can be found on the internet. When we ran the analysis with these data in hand, the result was: don't buy lottery tickets. It's too risky. Even the enormous returns we found are not big enough to counteract the enormous likelihood of not winning.

(As the lottery ads remind you, you can't win if you don't play. To this we say, The same argument applies to investing in building a time machine, yet no one seems to think that that's a good idea.)

In the end, everybody was right—sort of. Our instincts said we shouldn't invest in lottery tickets. If lottery tickets always had a negative rate of return, then obviously they would be a bad investment. But this is not the case; the mathematics showed that lottery tickets can have an excellent rate of return. As a result, we were forced to use some economic theory, which provided the rigorous justification we sought. Our instincts were right.